Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department
2^{nd} Year electrical power

Final Term Exam
Date: 11^{th} of June 2012
Mathematics 3(B) Code: EMP 272
Duration : 3 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:4
- Total Mark: 80 Marks

1- Solve the system $x + 4y + z = 2$, $4x + y + z = 5$, $x + y + 4z = 3$ using:

i) An iterative method ii) Cholesky decomposition iii) Gauss Jordan method

20 Marks

2- i) **Find** y(1.1) using **modified Euler** method for the differential equation:

$$x' = x^2 - 2tx + y - 2t,\quad y' = y - x^2 - 2tx + 2t + 3,\quad x(1) = 2,\quad y(1) = 3,\quad h = 0.1$$

ii) $x' = -10(x-y)$, $y' = -xz + 28x - y$, $z' = xy - 8z/3$, $x(0) = 2$, $y(0) = -1$, $z(0) = 3$, $h = 0.05$

Solve the above system using **Picard** method and find x(0.1) using **Euler** method.

20 Marks

3-i) Consider the problem of determining the steady state heat distribution in a thin square metal plate with dimensions 0.5m by 0.5m. Two adjacent boundaries are held at $0^\circ c$ and the heat of the other boundaries increases linearly from $0^\circ c$ at one corner to $100^\circ c$ where the sides meet. The problem is expressed as $\mathbf{u_{xx} + u_{yy} = 10x}$. If the grid is divided into 5 equal parts, find u(x,y) such that k = 0.25. Solve the constructed linear system of equations using Gauss elimination method.

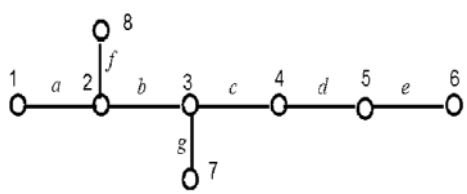
ii) Find the constants of the curve $\mathbf{y = a\cos x + b\ln x + c\,e^{x/10}}$ that fit **(1,3), (5,14), (19,101)**

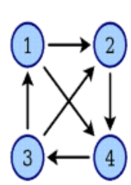
20 Marks

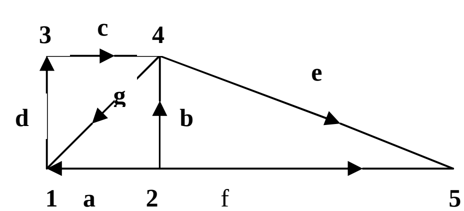
4-i) **Define** with an **example** for each of the following terms:

Simple Graph – Valency – Walk - Trail – Path - Complete Graph - Null Graphs - Bipartite Graphs - Tree Graph - Spanning Tree - Connected Graphs - Multi Graphs- Eulerian circuit – Eulerian path - Hamiltonian path -

ii) **Find** incidence and adjacency matrices for the following graphs







20 Marks

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Modified Euler states: $y_{i+1} = y_i + (h/2)[f(x_i, y_i) + f(x_{i+1}, y_i + hf(x_i, y_i))]$

Model answer

1-i) Using Gauss–Seidel:

Rarrange: $4x + y + z = 5$, $x + 4y + z = 2$, $x + y + 4z = 3$

$x^{(k+1)} = [5 - y^{(k)} - z^{(k)}]/4$, $y^{(k+1)} = [2 - x^{(k+1)} - z^{(k)}]/4$, $z^{(k+1)} = [3 - x^{(k+1)} - y^{(k+1)}]/4$

Let $(x,y,z)^{(0)} = (0,0,0)$, therefore the 1st iteration will be: $x^{(1)} = [5 - y^{(0)} - z^{(0)}]/4 = 1.25$,

$y^{(1)} = [2 - x^{(1)} - z^{(0)}]/4 = 0.1875$, $z^{(1)} = [3 - x^{(1)} - y^{(1)}]/4 = 0.3906$ and the 2nd iteration will be

$x^{(2)} = [5 - y^{(1)} - z^{(1)}]/4 = 1.1055$, $y^{(2)} = [2 - x^{(2)} - z^{(1)}]/4 = 0.126$, $z^{(2)} = [3 - x^{(2)} - y^{(2)}]/4 = 0.4421$

1-ii)Using Cholesky method

$$\begin{pmatrix} 2 & 0 & 0 \\ 0.5 & 1.9365 & 0 \\ 0.5 & 0.3873 & 1.8974 \end{pmatrix}\begin{pmatrix} 2 & 0.5 & 0.5 \\ 0 & 1.9365 & 0.3873 \\ 0 & 0 & 1.8974 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0.5 & 1.9365 & 0 \\ 0.5 & 0.3873 & 1.8974 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}, \text{ therefore } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2.5 \\ 0.3873 \\ 0.8433 \end{pmatrix}$$

Also $$\begin{pmatrix} 2 & 0.5 & 0.5 \\ 0 & 1.9365 & 0.3873 \\ 0 & 0 & 1.8974 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2.5 \\ 0.3873 \\ 0.8433 \end{pmatrix}, \text{ therefore } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.1111 \\ 0.1111 \\ 0.4445 \end{pmatrix}$$

1-iii) Using Gauss Jordan method:

$$\left(\begin{array}{ccc|c} 4 & 1 & 1 & 5 \\ 1 & 4 & 1 & 2 \\ 1 & 1 & 4 & 3 \end{array}\right) \approx \left(\begin{array}{ccc|c} 4 & 1 & 1 & 5 \\ 0 & -15 & -3 & -3 \\ 0 & -3 & -15 & -7 \end{array}\right) \approx \left(\begin{array}{ccc|c} 60 & 0 & 12 & 72 \\ 0 & -15 & -3 & -3 \\ 0 & 0 & 72 & 32 \end{array}\right) \approx \left(\begin{array}{ccc|c} 360 & 0 & 0 & 400 \\ 0 & -360 & 0 & -40 \\ 0 & 0 & 72 & 32 \end{array}\right)$$

Therefore $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10/9 \\ 1/9 \\ 4/9 \end{pmatrix}$$

2-i) $\mathbf{x' = x^2 - 2tx + y - 2t = f(x,y,t)}$, $\mathbf{y' = y - x^2 - 2tx + 2t + 3 = \varphi(x,y,t)}$, $x_0 = 2$, $y_0 = 3$, $t_0 = 1$

$$y_{i+1} = y_i + (h/2)[\varphi(t_i, x_i, y_i) + \varphi(t_{i+1}, x_i + hf(t_i, x_i, y_i), y_i + h\varphi(t_i, x_i, y_i))]$$

Put i = 0, therefore

$$y_1 = y_0 + (h/2)[\varphi(t_0, x_0, y_0) + \varphi(t_1, x_0 + hf(t_0, x_0, y_0), y_0 + h\varphi(t_0, x_0, y_0))] = 2.9585$$

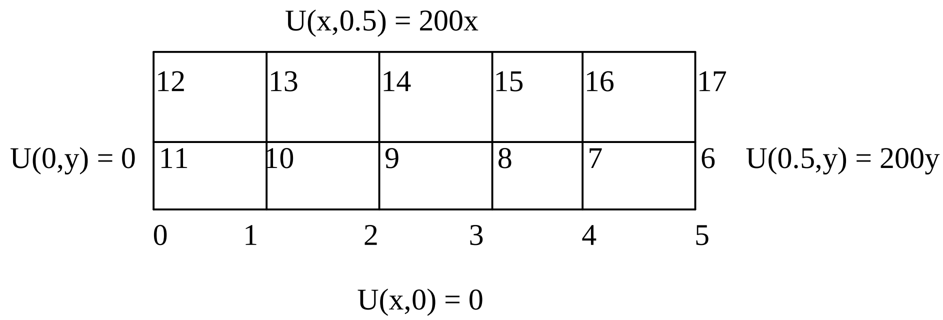
2-ii) $y_{n+1} = y_0 + \int_{t_0}^{t} (x_n z_n + 28x_n - y_n)\,dt$, $x_{n+1} = x_0 + \int_{t_0}^{t} -10(x_n - y_n)\,dt$,

$z_{n+1} = z_0 + \int_{t_0}^{t} (x_n y_n - 8z_n/3)\,dt$, $y_0 = -1$, $x_0 = 2$, $t_0 = 0$, $z_0 = 3$, thus $x_1 = x_0 + \int_{t_0}^{t} -10(x_0 - y_0)\,dt$,

$y_1 = y_0 + \int_{t_0}^{t} (x_0 z_0 + 28x_0 - y_0)\,dt$ and $z_1 = z_0 + \int_{t_0}^{t} (x_0 y_0 - 8z_0/3)\,dt$, therefore $x_1 = 2-30t$, $y_1=-1+51t$, $z_1 = 3-10t$. Similarly, $x_2=x_0+\int_{t_0}^{t} -10(x_1 - y_1)\,dt$, $y_2 = y_0 + \int_{t_0}^{t} (x_1 z_1 + 28x_1 - y_1)\,dt$ and $z_2= z_0 + \int_{t_0}^{t} (x_1 y_1 - 8z_1/3)\,dt$, therefore $x_2 = 2 - 30t + 405t^2$, $y_2 =-1+51t-(781/2)t^2 - 100t^3$, $z_2 = 3-10t +(238/3)t^2 - 510t^3$.

2nd : <u>using Euler,</u> $x_{n+1} = x_n + h\,[-10(x_n - y_n)]$, $y_{n+1} = y_n + h\,[-x_n z_n + 28 x_n - y_n]$, thus $x_1 =x_0 +h[-10(x_0 - y_0)] = 0.5= x(0.05)$, $y_1 = y_0 +h\,[-x_0 z_0 + 28 x_0 - y_0] =1.55 = y(0.05)$, therefore $x(0.1) = x_2 = x_1 +h[-10(x_1 - y_1)] = 1.025$

3-i)



$u_0 = u_1 = u_2 = u_3 = u_4 = u_5 = u_{11} = u_{12} =0$, $u_6 =50$, $u_{17} =100$, $u_{16} =80$, $u_{15} = 60$, $u_{14} =40$, $u_{13} =20$

The formula of Poisson equation is simplified to:

$$\mathbf{0.0625[\, U_{i+1,j} + U_{i-1,j}\,] + 0.01[\, U_{i,j+1} + U_{i,j-1}\,] - 0.00625\, x_i = 0.145 U_{i,j}}$$

From which the following system of equations are constructed:

$0.0625\, U_8 - 0.145\, U_7 = -3.9225$, $0.0625[\, U_7 + U_9\,] - 0.145\, U_8 = -0.59813$,

$0.0625[\, U_8 + U_{10}\,] - 0.145\, U_9 = -0.39875$, $0.0625\, U_9 - 0.145\, U_{10} = -0.1994$,

3-ii) To get constants a,b,c, we have to use Least square method Such that

$$\sum_{i=1}^{3} y_i \cos(x_i) = a\sum_{i=1}^{3}[\cos(x_i)]^2 + b\sum_{i=1}^{3}[\cos(x_i)][\ln(x_i)] + c\sum_{i=1}^{3}[\cos(x_i)][e^{-x_i/10}]$$

$$\sum_{i=1}^{3} y_i\,[\ln(x_i)] = a\sum_{i=1}^{3}[\cos(x_i)][\ln(x_i)] + b\sum_{i=1}^{3}[\ln(x_i)]^2 + c\sum_{i=1}^{3}[\ln(x_i)][e^{-x_i/10}]$$

$$\sum_{i=1}^{3} y_i\,[e^{-x_i/10}] = a\sum_{i=1}^{3}[\cos(x_i)][e^{-x_i/10}] + b\sum_{i=1}^{3}[\ln(x_i)][e^{-x_i/10}] + c\sum_{i=1}^{3}[e^{-x_i/5}]$$

$$\sum_{i=1}^{3}[\cos(x_i)]^2 = 1.3499,\ \sum_{i=1}^{3}[\cos(x_i)][\ln(x_i)] = 3.3677,\ \sum_{i=1}^{3}[\cos(x_i)][e^{-x_i/10}] = 7.6751$$

$$\sum_{i=1}^{3}[\ln(x_i)]^2 = 11.2597,\ \sum_{i=1}^{3}[\ln(x_i)][e^{-x_i/10}] = 22.3394 \text{ and } \sum_{i=1}^{3}[e^{-x_i/5}] = 48.641,$$ from which we can get a, b, c

4-i) **Simple Graph**: A graph with no loops or multiple edges is called a simple graph
Valency:Is the degree of vertices
Walk: Pass through vertices and edges of the graph and may pass through repeated vertices and edges
Trail: If all the edges (but no necessarily all the vertices) of a walk are different, then the walk is called a trail (i.e. walk with no repeated edges)
Path: All edges and vertices of walk are different, then the trail is called path(i.e. trail with no repeated vertices) .
Complete Graphs: Is a graph in which every two distinct vertices are joined by exactly one edge
Null Graphs: graph containing no edges
Bipartite Graphs: Is a graph whose vertex-set can be split into two sets in such a way that each edge of the graph joins a vertex in first set to a vertex in second set.

Tree Graph: A tree is a connected graph which has no cycles.

Spanning Tree: If G is a connected graph, the spanning tree in G is a sub graph of G which includes every vertex of G and is also a tree.

Connected Graphs: A graph G is connected if there is a path in G between any given pair of vertices, otherwise it is disconnected

Multi Graphs: A multigraph or pseudograph is a graph which is permitted to have multiple edges
Eulerian circuit: Is a Eulerian trail which starts and ends on the same vertex
Eulerian path: Is a trail in a graph which visits every edge exactly once.

Hamiltonian path: Is a path in a graph G that passes through every vertex exactly once.

Incidence matrices:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},\ \begin{pmatrix} -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 & 0 & 0 \end{pmatrix},\ \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Adjacency matrices:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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2^{nd} Year Mechanical power

Final Term Exam
Date: 4^{th} of June 2012
Mathematics & Computer (B)
Code: MDE 232
Duration : 3 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:3
- Total Mark: 70 Marks

1-a) **Solve** the following system using **an iterative method** starting $[x_2, x_3, x_4]$ = **[-1, 1, 1]**

$$2x_1 + 8x_2 + 3x_3 + x_4 = -2, \quad 7x_1 - 2x_2 + x_3 + 2x_4 = 3, \quad 2x_2 - x_3 + 4x_4 = 4, \quad -x_1 + 5x_3 + 2x_4 = 5$$

[12 marks]

1b-i) Make C^{++} program to write the above system of linear equations. **[4 marks]**

1b-ii) Discuss the basic data types in C^{++} program and the function that must be present in C programs. **[4 marks]**

2-a) Solve the above system using LU decomposition (**diagonal elements of L are unity**)

[10 marks]

2b-i) Solve the following system of equations using Picard up to 2^{nd} approximation

$$x` - 3y` = -2t + x - 2y - 7, \quad 2x` + y` = 10t + y + 3 - t^2, \quad x(0) = 1, \quad y(0) = -3$$ **[7 marks]**

Find y(0.1) using **Euler,** given h = 0.05 **[6 marks]**

2b-ii) Find y(0.1) using **Runge-Kutta method of order four** for the differential equation

$$y` - y = -0.5\, e^{t/2} \sin(5t) + 5\, e^{t/2} \cos(5t), \quad y(0) = 0, \quad h = 0.1$$ **[7 marks]**

3-a) Consider elliptic equation $U_{xx} + U_{yy} = xe^{y}$, with B.C. $U(0,y) = y$, $U(2,y) = e^{2y}$, $0 \le y \le 1$ & $U(x,0) = x/2$, $U(x,1) = e^{x}$, $0 \le x \le 2$. Find U(x,y) of the grid points using **Gauss-Jordan** method to solve the linear system of equations given **h = 0.5, k = 0.2** **[12 marks]**

3-b) Consider wave equation $u_{tt} = 4u_{xx}$, $0 < x < 1$, $0 < t$ with B.C. $U(0,t) = U(1,t) = 0$, I.C. $U(x,0) = \sin \pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$, **h = 0.2, k = 0.0005**. Find U(x,t) of the grid points. **[8 marks]**

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Runge-Kutta method of order four states: $y_{i+1} = y_i + (1/6)[k_1 + 2k_2 + 2k_3 + k_4]$,
$k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + h/2, y_i + k_1/2)$, $k_3 = hf(x_i + h/2, y_i + k_2/2)$, $k_4 = hf(x_{i+1}, y_i + k_3)$

Model answer

1-a) Rarrange: $7x_1 - 2x_2 + x_3 + 2x_4 = 3$, $2x_1 + 8x_2 + 3x_3 + x_4 = -2$, $-x_1 + 5x_3 + 2x_4 = 5$, $2x_2 - x_3 + 4x_4 = 4$.

$x_1^{(k+1)} = [3 + 2x_2^{(k)} - x_3^{(k)} - 2x_4^{(k)}]/7$, $x_2^{(k+1)} = [-2 - 2x_1^{(k+1)} - 3x_3^{(k)} - x_4^{(k)}]/8$, $x_3^{(k+1)} = [5 + x_1^{(k+1)} - 2x_4^{(k)}]/5$, $x_4^{(k+1)} = [4 - 2x_2^{(k+1)} + x_3^{(k+1)}]/4$

Since $[x_1, x_2, x_3, x_4] = [0,-1,1,1]$, therefore the 1st iteration will be:

$x_1^{(1)} = [3 + 2x_2^{(0)} - x_3^{(0)} - 2x_4^{(0)}]/7 = -0.2857$, $x_2^{(1)} = [-2 - 2x_1^{(1)} - 3x_3^{(0)} - x_4^{(0)}]/8 = -0.6786$, $x_3^{(1)} = [5 + x_1^{(1)} - 2x_4^{(0)}]/5 = 0.5429$, $x_4^{(1)} = [4 - 2x_2^{(1)} + x_3^{(1)}]/4 = 0.3393$

and the 2nd iteration will be $x_1^{(2)} = [3 + 2x_2^{(1)} - x_3^{(1)} - 2x_4^{(1)}]/7 = 0.0602$, $x_2^{(2)} = [-2 - 2x_1^{(2)} - 3x_3^{(1)} - x_4^{(1)}]/8 = -0.5111$, $x_3^{(2)} = [5 + x_1^{(2)} - 2x_4^{(1)}]/5 = 0.8763$, $x_4^{(2)} = [4 - 2x_2^{(2)} + x_3^{(2)}]/4 = 0.91625$

1-b) Main( )

```
{ count<<"2X1 +8X2 + 3X3 +X4  = -2 ";
Count<<"7X1 – 2X2 + X3 +2X4 = 3 "<<endl;
Count<<"2X2 - X3 + 4X4  =  4";
Count<<"-X1 + 5X3 + 2X4 = 5";
}
```

2-a) Using LU decomposition:

$$\begin{pmatrix} 2 & 8 & 3 & 1 \\ 7 & -2 & 1 & 2 \\ 0 & 2 & -1 & 4 \\ -1 & 0 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3.5 & 1 & 0 & 0 \\ 0 & -0.0667 & 1 & 0 \\ -0.5 & -0.1333 & -3.2041 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 & 3 & 1 \\ 0 & -30 & -9.5 & -1.5 \\ 0 & 0 & -1.6333 & 3.9 \\ 0 & 0 & 0 & 14.7959 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 3.5 & 1 & 0 & 0 \\ 0 & -0.0667 & 1 & 0 \\ -0.5 & -0.1333 & -3.2041 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 4 \\ 5 \end{pmatrix}, \text{ therefore } \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 10 \\ 4.6667 \\ 20.2857 \end{pmatrix} \text{ and}$$

$$\begin{pmatrix} 2 & 8 & 3 & 1 \\ 0 & -30 & -9.5 & -1.5 \\ 0 & 0 & -1.6333 & 3.9 \\ 0 & 0 & 0 & 14.7959 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 10 \\ 4.6667 \\ 20.2857 \end{pmatrix}, \text{ therefore } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -0.1751 \\ -0.5338 \\ 0.4165 \\ 1.371 \end{pmatrix}$$

2b-i) Put $y` = z$, thus $z` = 2y - tz$, $y_0 = z_0 = 2$, $t_0 = 1$, therefore $y_0^{(1)} = z_0 = 2$ and $z_0^{(1)} = 2$,

also $y`` = z`$, then $y_0^{(2)} = 2$ and $z_0^{(2)} = 0 = y_0^{(3)}$, hence

$$y(x) = y_0 + \frac{t - t_0}{1!} y_0^{(1)} + \frac{(t - t_0)^2}{2!} y_0^{(2)} + \frac{(t - t_0)^3}{3!} y_0^{(3)} + \cdots$$

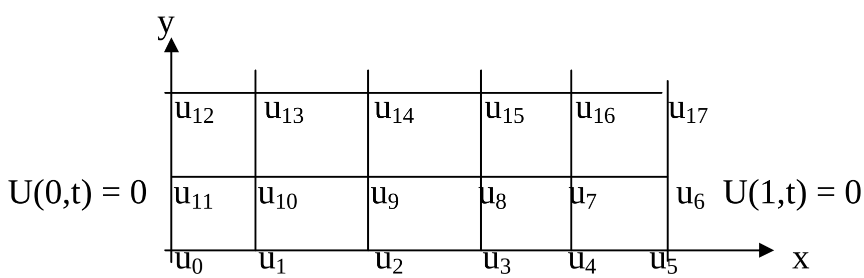
2b-ii) $y_{i+1} = y_i + (1/6)[k_1 + 2k_2 + 2k_3 + k_4]$, $y_0 = t_0 = 0$, $f(t,y) = -0.5\, e^{t/2} \sin(5t) + 5\, e^{t/2} \cos(5t) + y$

$k_1 = hf(t_i, y_i)$, $k_2 = hf(t_i+h/2, y_i+ k_1/2)$, $k_3 = hf(t_i+h/2, y_i+ k_2/2)$, $k_4 = hf(t_{i+1}, y_i+ k_3)$,

Therefore $k_1^{(0)} = 0.5$, $k_2^{(0)} = 0.509$, $k_3^{(0)} = 0.5096$, $k_4^{(0)} = 0.487$, thus

$y_1 = y_0 + [k_1^{(0)} + 2\, k_2^{(0)} + 2\, k_3^{(0)} + k_4^{(0)}] = y(0.1) = 0.504$

3-a) $u_{i,j+1} = \frac{k}{h^2} [u_{i+1,j} + u_{i-1,j}] + [1- \frac{2k}{h^2}] u_{i,j} = 5[u_{i+1,j} + u_{i-1,j}] - 9u_{i,j}$



$u_0 = u_5 = u_6 = u_{11} = u_{12} = u_{17} = 0$, $u_1 = 0.4$, $u_2 = 0.8$, $u_3 = 0.8$, $u_4 = 0.4$

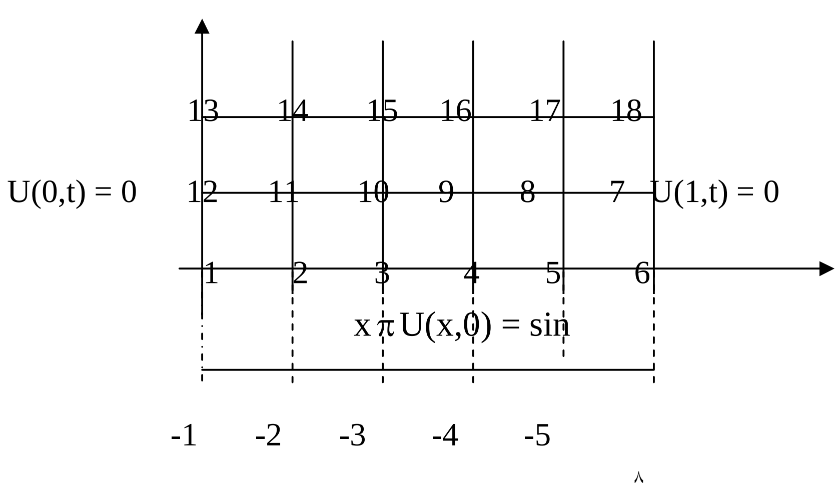
Therefore $u_{i,j+1} = 5[u_{i+1,j} + u_{i-1,j}] - 9u_{i,j}$, thus $u_7 = 5[u_5 + u_3] - 9u_4 = 0.4$,

$u_8 = 5[u_4 + u_2] - 9u_3 = -1.2$, $u_9 = 5[u_3 + u_1] - 9u_2 = -1.2$, $u_{10} = 5[u_2 + u_0] - 9u_1 = 0.4$

$u_{13} = 5[u_9 + u_{11}] - 9u_{10} = -9.6$, $u_{14} = 5[u_8 + u_{10}] - 9u_9 = 6.8$, $u_{15} = 5[u_7 + u_9] - 9u_8 = 6.8$

$u_{16} = 5[u_6 + u_8] - 9u_7 = -9.6$.

3-b)



At t = 0, $\frac{\partial u}{\partial t}(x,0) = \frac{u_{i,j} - u_{i,j-1}}{k}$ = 0, therefore $u_{i,j} = u_{i,j-1}$

Since u_1= 0, u_2 = sin(0.2 π)= sin(36) , u_3 = sin(0.4 π)= sin(72) , u_4 = sin(0.6 π)= sin(108), u_5 = sin(0.8 π) = sin(144) , u_6 = sin(π) = 0, u_{12} = u_{13} = u_7 = u_{18} = 0 and $\frac{p^2k^2}{h^2} = \frac{2^2(0.0005)^2}{(0.2)^2}$ = 0.000025. Therefore by referring to (3), we get

At Point 11:

u_{11} = 0.000025[u_3 + u_1] + 1.99995 u_2 – u_{-2}

At Point 10:

U_{10} = 0.000025[u_4 + u_2] + 1.99995 u_3 – u_{-3}

At Point 9:

U_9 = 0.000025[u_5 + u_3] + 1.99995 u_4 – u_{-4}

At Point 8:

U_8 = 0.000025[u_6 + u_4] + 1.99995 u_5 – u_{-5}